$y_1 = t$

$y = v y_1 = vt$

$y' = v't + vt' = v't + v$

$y'' = v''t + v' + v' = v''t + 2v'$

$v''t^4 + 2v't^3 - v't^2 - vt + vt = 0$

$v''[t^4] + v'[2t^3 - t^2] = 0$

let $w = v'$

$w't^4 + w(2t^3 - t^2) = 0$

$\frac{1}{w}\frac{dw}{dt} = \frac{t^2 - 2t^3}{t^4}$

$\int \frac{1}{w}\,dw = \int \frac{t^2 - 2t^3}{t^4}\,dt$

$\ln w = \int t^{-2} - 2t^{-1}\,dt = -t^{-1} - 2\ln t + C$

$w = e^{-1/t} e^{\ln t^{-2}} e^{C} = C t^{-2} e^{-1/t}$

$w = v' \rightarrow v = \int C t^{-2} e^{-1/t} = C_1 e^{-1/t} + C_2$

$v = y/t \rightarrow y = C_1 t e^{-1/t} + C_2 t$

given $y_1 = 2$

thus, $y_2 = te^{-1/t}$

$\boxed{te^{-1/t}}$

Given that $y_1 = t$ is a solution of the following equation:

$$t^3 y'' - ty' + y = 0.$$

Which of the following is also a solution?

Ⓐ $t^2 e^{-1/t}$

Ⓑ $t\ln(t)$

Ⓒ $t\ln^2(t)$

Ⓓ $t^2 e^t$

Ⓔ $te^{-1/t}$

$$y = vy_1 = vx^5$$
$$y' = v'x^5 + 5vx^4$$
$$y'' = v''x^5 + 5v'x^4 + 5v'x^4 + 20vx^3$$
$$x^2\left[v''x^5 + 10v'x^4 + 20vx^3\right] - x\left[v'x^5 + 5vx^4\right] - 15vx^5$$
$$v''x^7 + 10v'x^6 + 20vx^5 - v'x^6 - 5vx^5 - 15vx^5$$
$$v''x^7 + 9v'x^6 = 0$$
let $w = v'$
$$w'x^7 + w\,9x^6 = 0$$
$$\frac{1}{w}\,dw = -9x^{-1}\,dx$$
$$\ln w = -9\ln x + C$$
$$w = Cx^{-9}$$
$$v = \int Cx^{-9} = -\frac{1}{8}C_1x^{-8} + C_2$$
$$v = \frac{y}{y_1} \rightarrow y_2 = -\frac{1}{8}C_1x^{-3} + C_2x^5$$
$$\boxed{x^{-3}}$$

It is known that one of the solutions of the differential equation

$$x^2y'' - xy' - 15y = 0 \quad (x > 0)$$

is $y_1(x) = x^5$. Use the method of reduction of order to find a second linearly independent solution $y_2(x)$. Recall that this method consists of substituting $y_2(x) = v(x)y_1(x)$ into the differential equation above and reducing it to a first order equation for v'.