$$dy/dx + P(x)y = Q(x)$$

$$\rho(x) = e^{\int P(x)\,dx}$$

$$y' + 2x^{-1}y = e^{3x}x^{-2}$$

$$\rho(x) = e^{\int 2x^{-1}} = e^{2\ln|x|} = x^2$$

$$y(x) = \frac{1}{\rho(x)}\int \rho(x)Q(x)\,dx$$

$$= \frac{1}{x^2}\int e^{3x}\,dx = \frac{e^{3x}}{3x^2} + \frac{c}{x^2}$$

The general solution to $x^2y' + 2xy = e^{3x}$ is

- Ⓐ $y = \frac{3}{x^2}e^{3x} + c$
- Ⓑ $y = ce^{3x}$
- Ⓒ $y = \frac{1}{3x^2}e^{3x} + cx^{-2}$
- Ⓓ $y = \frac{1}{2x^2}e^{3x}$
- Ⓔ $y = \frac{1}{3x}e^{3x} + cx^{-2}$

$p(x) = e^{\int 3\,dx} = e^{3x}$

$y(x) = \frac{1}{p(x)} \int Q(x)\,p(x)\,dx$

$= e^{-3x} \int 3x^2 + 2x\,dx$

$= e^{-3x}x^3 + e^{-3x}x + Ce^{-3x}$

Find a general solution to the following differential equation

$$\frac{dy}{dx} + 3y = 3x^2e^{-3x} + 2xe^{-3x}$$

Ⓐ $y = (x^3 + x^2)e^{-3x} + C$

Ⓑ $y = (x^3 + x^2 + C)e^{-3x}$

Ⓒ $y = (3x^2 + 2x + C)e^{-3x}$

Ⓓ $y = -\left(x^2 + \frac{4}{3}x + \frac{4}{9}\right)e^{-3x} + C$

Ⓔ $y = (x^3 + x^2)e^{-3x} + Ce^{3x}$

$$y' - 3x^{-1}y = 2x^3$$
$$p(x) = e^{\int -3x^{-1}dx} = e^{-3\ln x} = x^{-3}$$
$$y(x) = x^3 \int 2\, dx = (2x + C)x^3$$
$$y(x) = 2x^4 + Cx^3$$
$$y(1) = 0 = 2 + C \rightarrow C = -2$$
$$y(2) = 32 - 16 = 16$$

Let $y(x)$ be the solution to the initial value problem

$$xy' = 3y + 2x^4, \quad y(1) = 0.$$

Then, $y(2)$ is

Ⓐ 8

Ⓑ 4

Ⓒ 20

Ⓓ 16

Ⓔ 32

If $y(x)$ is a solution to the initial value problem

$$\frac{dy}{dx} + \frac{y}{2+2x} = 6, \quad y(0) = 2$$

then $y(3) = ?$

- Ⓐ 12
- Ⓑ -17
- Ⓒ 28
- Ⓓ 15
- Ⓔ -23

$$y' + \frac{y}{2+2x} = 6$$

$$p = e^{\int \frac{1}{2+2x}dx} = e^{\int \frac{1}{2}u^{-1}du} = e^{\frac{1}{2}\ln u}$$

$$= e^{\frac{1}{2}\ln(2+2x)}$$

$$= e^{\ln((2+2x)^{1/2})}$$

$$= (2+2x)^{1/2}$$

$u = 2+2x$

$du/dx = 2$

$dx = \frac{1}{2}du$

$$y(x) = \frac{1}{p(x)}\int Q(x)\,p(x)\,dx$$

$$= \frac{1}{(2+2x)^{1/2}}\int 6(2+2x)^{1/2}\,dx$$

$u = 2+2x$

$dx = \frac{1}{2}du$

$$= (2+2x)^{-1/2}\int 3u^{1/2}\,dx = (2+2x)^{-1/2}\left[2u^{3/2} + C\right]$$

$$= (2+2x)^{-1/2}\left[2(2+2x)^{3/2} + C\right]$$

$$y(x) = 2(2+2x) + C(2+2x)^{-1/2}$$

$$y(0) = 2 = 2(2+2(0)) + C(2+2(0))^{-1/2}$$

$$2 = 4 + \frac{C}{\sqrt{2}}$$

$$2\sqrt{2} = 4\sqrt{2} + C \rightarrow C = -2\sqrt{2}$$

$$y(3) = 2(2+2(3)) + (2+2(3))^{-\frac{1}{2}}(-2\sqrt{2})$$

$$= 16 + \frac{1}{2\sqrt{2}}(-2\sqrt{2}) = 15$$

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

$$v = y^{1-n}$$

If y is a solution to the initial value problem for a Bernoulli equation

$$\frac{dy}{dx} + \frac{4y}{x} = 3y^2, \quad y(1) = \frac{1}{2},$$

then $y(2) = ?$ Hint: Use the substitution $v = y^{-1}$.

- (A) $\frac{1}{18}$
- (B) $\frac{1}{34}$
- (C) $\frac{1}{10}$
- (D) $\frac{1}{20}$
- (E) $\frac{1}{32}$

$$v = y^{1-n} = y^{-1}$$

$$\frac{dv}{dx} = -y^{-2}\, dy/dx$$

$$\boxed{\frac{dy}{dx} y^{-2}} + 4x^{-1}y^{-1} = 3$$

$$-\frac{dv}{dx} + 4x^{-1}v = 3 \rightarrow \frac{dv}{dx} - 4x^{-1}v = -3$$

$$\rho(x) = e^{\int -4x^{-1}dx} = e^{-4\ln x} = \cancel{e}^{\cancel{\ln}(x^{-4})} = x^{-4}$$

$$v(x) = \frac{1}{\rho(x)} \int Q(x)\,\rho(x)\,dx$$

$$v(x) = x^4 \int -3x^{-4}\,dx = \left(x^{-3} + C\right)x^4$$

$$v(x) = x + Cx^4$$

$$y^{-1} = x + Cx^4$$

$$y(x) = \frac{1}{x + Cx^4}$$

$$y(1) = \frac{1}{2} = \frac{1}{1+C} \rightarrow C = 1$$

$$y(2) = \frac{1}{2+16} = \frac{1}{18}$$

Find the general solution of the equation

$$(t^2+1)\frac{dy}{dt} + 4ty = 1$$

Ⓐ $y = \dfrac{\frac{1}{2}t^2 + C}{t^2+1}$

Ⓑ $y = (t^2+1)(\tan^{-1}(t) + C)$

Ⓒ $y = e^{-4t}(\tan^{-1}(t) + C)$

Ⓓ $y = \dfrac{\frac{1}{3}t^3 + t + C}{(t^2+1)^2}$

Ⓔ $y = 2\ln(t^2+1) + C$

$$y' + \frac{4t}{t^2+1}y = \frac{1}{t^2+1}$$

$$p(t) = e^{\int \frac{4t}{t^2+1}dt} = e^{\int \frac{4\cancel{t}}{u}\frac{1}{2\cancel{t}}du} = \int \frac{2}{u}du$$

$u = t^2+1$

$du/dt = 2t$

$dt = \frac{1}{2t}du$

$$= e^{2\ln u} = e^{\ln u^2} = u^2 = (t^2+1)^2$$

$$y(x) = \frac{1}{p(t)}\int Q(t)\,p(t)\,dt$$

$$= \frac{1}{(t^2+1)^2}\int (t^2+1)^2\frac{1}{t^2+1}dt$$

$$= \frac{1}{(t^2+1)^2}\int t^2+1\,dt = \frac{1}{(t^2+1)^2}\left[\frac{1}{3}t^3 + t + C\right]$$

A ball with mass 0.2 kg is thrown upward with initial velocity 49 m/s from the ground. There is a force due to air resistance of magnitude |v|/25 directed opposite to the velocity v (measured in m/s). How much time does it take for the ball to reach its maximum height? (Use that the gravitation acceleration $g = 9.8\ m/s^2$.)

$$F = -mg - \frac{1}{25}v$$

$$F = ma = m\,dv/dt$$

$$-mg - \frac{1}{25}v = m\,dv/dt$$

$$-0.2(9.8) - \frac{v}{25} = 0.2\,dv/dt$$

$$-\frac{1}{5}\,dv/dt - \frac{v}{25} = \frac{1}{5}(9.8)$$

$$dv/dt + \frac{1}{5}v = -9.8$$

① $y' + P(x)y = Q(x)$

② $\rho(x) = e^{\int P(x)\,dx}$

③ $y(x) = \frac{1}{\rho(x)}\int Q(x)\,\rho(x)\,dx$

$$\rho(t) = e^{\int \frac{1}{5}dt} = e^{1/5\,t}$$

$$v(t) = e^{-1/5\,t}\int -9.8\,e^{1/5\,t}\,dt = e^{-1/5\,t}\left[-49\,e^{1/5\,t} + C\right]$$

$$v(t) = -49 + Ce^{-1/5\,t}$$

$$v(0) = -49 + Ce^{0} = 49 \rightarrow C = 98$$

$$v(t) = -49 + 98e^{-t/5}$$

$$0 = -49 + 98e^{-t/5}$$

$$1/2 = e^{-t/5}$$

$$\ln\frac{1}{2} = -\frac{t}{5}$$

$$t = -5\ln\frac{1}{2} = -5(\ln 1 - \ln 2) = 5\ln 2 = \ln 32$$

$$W(f,g) = \det \begin{vmatrix} f(t) & g(t) \\ f'(t) & g'(t) \end{vmatrix}$$

If the Wronskian $W(f,g) = 4t^4$ and $f(t) = t^2$, for $t > 0$, then the function g could be:

- Ⓐ $Ct + 4t^4$
- Ⓑ $Ct^2 + 4t^2$
- Ⓒ $Ct^2 + 4t^4$
- Ⓓ $Ct + 4t^3$
- Ⓔ $Ct^2 + 4t^3$

$$4t^4 = \det \begin{vmatrix} t^2 & g(t) \\ 2t & g'(t) \end{vmatrix} = t^2 g' - 2tg$$

$$t^2 g' - 2tg = 4t^4$$

$$g' - 2t^{-1} g = 4t^2$$

$y' + P(x)y = Q(x) \rightarrow$ easy to solve with integrating factor

$$\rho(t) = e^{\int -2t^{-1}dt} = e^{-2\ln t} = t^{-2}$$

$$g(t) = t^2 \int (4t^2)(t^{-2})\,dt = t^2 \left[4t + C \right] = 4t^3 + Ct^2$$

$y' - x^{-1}y = x^3$

$p(x) = e^{\int -x^{-1}dx} = e^{-\ln x} = x^{-1}$

$y(x) = x\int x^2 dx = x\left[\frac{1}{3}x^3 + C\right] = \frac{x^4}{3} + Cx$

$y(1) = \frac{1}{3} + C = \frac{4}{3} \rightarrow C = 1$

$y(3) = \frac{3^4}{3} + x = 3^3 + 3 = 30$

If y is a solution to the the initial value problem

$$\frac{dy}{dx} - \frac{y}{x} = x^3, \quad y(1) = \frac{4}{3}$$

then $y(3) = ?$

- Ⓐ 20
- Ⓑ 4
- Ⓒ 27
- Ⓓ 30
- Ⓔ $\frac{8}{3}$

$$y' + 5x^{-1}y = e^{4x}x^{-5}$$

$$p(x) = e^{\int 5x^{-1}\,dx} = e^{5\ln x} = x^5$$

$$y(x) = x^{-5}\int e^{4x}\,dx$$

$$= x^{-5}\left[\frac{1}{4}e^{4x} + C\right]$$

The general solution $y(x)$ to

$$x^2y' + 5xy = \frac{e^{4x}}{x^3}, \quad x > 0$$

is

Ⓐ $y(x) = \frac{e^{4x}}{x^3} + C$

Ⓑ $y(x) = \frac{e^{4x}}{4x^5} + \frac{C}{x^5}$

Ⓒ $y(x) = \frac{e^{x}}{x^3} + \frac{C}{x^2}$

Ⓓ $y(x) = Cx^{-5}$

Ⓔ $y(x) = \frac{Ce^{4x}}{4}$

$$y' + 2e^{-x}y = 2e^{2e^{-x}}$$

$$p(x) = e^{\int 2e^{-x}dx} = e^{-2e^{-x}}$$

$$y(x) = e^{2e^{-x}}\int 2\,dx$$

$$e^{2e^{-x}}[2x + C]$$

$$y(x) = 2xe^{2e^{-x}} + Ce^{2e^{-x}}$$

Find the general solution to the differential equation

$$e^x\frac{dy}{dx} + 2y = 2e^{x+2e^{-x}}.$$

Ⓐ $y = x^2e^{2e^{-x}} + Ce^{2e^{-x}}$

Ⓑ $y = xe^{x+2e^{-x}} + Ce^{x+2e^{-x}}$

Ⓒ $y = x^2e^x + Ce^x$

Ⓓ $y = x^2e^{-x} + Ce^{x+2e^{-x}}$

Ⓔ $y = 2xe^{2e^{-x}} + Ce^{2e^{-x}}$