$$r^3 + 3r^2 + 2r = 0$$

$$r(r^2 + 3r + 2) = 0$$

$$r = 0, -1, -2$$

$$y(t) = A + Be^{-t} + Ce^{-2t}$$

$$y'(t) = -Be^{-t} - 2Ce^{-2t}$$

$$y''(t) = Be^{-t} + 4Ce^{-2t}$$

$$y(0) = A + B + C = 2$$

$$y'(0) = -B - 2C = -1$$

$$y''(0) = B + 4C = 1$$

$$1 - 2C + 4C = 1 + 2C = 1$$

$$2C = 0$$

$$C = 0$$

$$-B = -1 \rightarrow B = 1$$

$$A = 1$$

$$y(t) = 1 + e^{-t}$$

$$y(1) = 1 + e^{-1}$$

Let $y(t)$ denote the unique solution to the initial value problem

$$y''' + 3y'' + 2y' = 0 \quad y(0) = 2, \quad y'(0) = -1, \quad y''(0) = 1.$$

What is the value of $y(1)$?

- Ⓐ $1 + 2e^{-1} + e^{-2}$
- Ⓑ $1 - e^{-2}$
- Ⓒ $1 + e^{-1}$
- Ⓓ $e + e^2$
- Ⓔ $2 + e^{-1} + 2e^{-2}$

$$F(t) = mx'' + Cx' + Kx$$
$$0 = 4x'' + 4x' + 17x$$

$$4r^2 + 4r + 17 = 0$$
$$r^2 + r + \frac{17}{4} = 0$$
$$r^2 + r + \frac{1}{4} + 4 = 0$$
$$\left(r + \frac{1}{2}\right)^2 + 4 = 0$$
$$\left(r + \frac{1}{2}\right)^2 = -4$$
$$r + 1/2 = \pm 2j$$
$$r = \pm 2j - 1/2$$
$$x(t) = e^{-\frac{1}{2}t}\left(A\cos 2t + B\sin 2t\right)$$
$$x'(t) = -\frac{1}{2}e^{-1/2t}\left(A\cos 2t + B\sin 2t\right) + e^{-\frac{1}{2}t}\left(-2A\sin 2t + 2B\cos 2t\right)$$

$$x(0) = A\cos 0 + B\sin 0 = A = 0$$
$$x'(0) = -\frac{1}{2}(0) + 2B = 2 \rightarrow B = 1$$

$$x_p(2) = \frac{\sin 4}{e}$$

A mass-spring-dashpot system with mass m=4, damping constant c=4, and spring constant k=17 is set in free motion with initial conditions x(0)=0 and x'(0)=2, where x(t) is the displacement from the equilibrium position at time t. Find x(2).

- Ⓐ $e^{-1}\sin(4)$
- Ⓑ $e^{-1}\cos(4)$
- Ⓒ $e^{-1}\cos(4) + e^{-1}\sin(4)$
- Ⓓ $e^{-2}\cos(1) + e^{-2}\sin(1)$
- Ⓔ $e^{-2}\sin(1)$

$y(t) = Ae^{4t} + Bte^{4t} + C$

$r = 4\ m2,\ 0$

$r(r-4)^2$

$= r(r^2 - 8r + 16)$

$= r^3 - 8r^2 + 16r$

$y''' - 8y'' + 16y' + C = 0$

Which of the following differential equations has

$$y(t) = C_1e^{4t} + C_2te^{4t} + C_3$$

as a general solution?

- Ⓐ $y'' - 8y + 16y = 0$
- Ⓑ $y^{(3)} - 8y'' + 16y' + y = 0$
- Ⓒ $y^{(3)} - 8y'' + 16y' = 0$
- Ⓓ $y^{(3)} - 8y'' + 24y' - 16y = 0$
- Ⓔ $y^{(3)} - 16y' = 0$

$$r^2 - 10r + 26 = 0$$

$$r^2 - 10r + 25 + 1 = 0$$

$$(r-5)^2 = -1$$

$$r - 5 = \pm j$$

$$r = 5 \pm j$$

$$y(t) = e^{5t}(A\cos t + B\sin t)$$

$$y'(t) = 5e^{5t}(A\cos t + B\sin t) + e^{5t}(-A\sin t + B\cos t)$$

$$y(0) = A = 1$$

$$y'(0) = 5 + B = 4 \rightarrow B = -1$$

$$y(t) = e^{5t}(\cos t - \sin t)$$

Solve the initial value problem

$$y'' - 10y' + 26y = 0, y(0) = 1, \\ y'(0) = 4$$

Ⓐ $y(t) = e^t(\cos 5t - \frac{2}{5}\sin 5t)$

Ⓑ $y(t) = e^{5t}(\cos t + 4\sin t)$

Ⓒ $y(t) = e^t(\cos 5t + \frac{3}{5}\sin 5t)$

Ⓓ $y(t) = e^t(\cos t + 2\sin t)$

Ⓔ $y(t) = e^{5t}(\cos t - \sin t)$

$r^3 + 6r^2 + 9r = 0$

$r(r+3)^2 = 0$

$r = -3\ m2,\ 0$

$y(t) = A + Be^{-3t} + Cte^{-3t}$

Find the general solution of $y''' + 6y'' + 9y' = 0$

A. $y(t) = c_1 + c_2 e^{-3t} + c_3 t e^{-3t}$

B. $y(t) + c_1 t + c_2 e^{3t} + c_3 t e^{3t}$

C. $y(t) = c_1 + c_2 \cos(-3t) + c_2 \sin(-3t)$

D. $y(t) = c_1 t + c_2 e^{-3t} + c_3 e^{3t}$

E. $y(t) = c_1 t + c_2 \cos(3t) + c_2 \sin(3t)$

$r^2 + r - 12 = 0$

$r = 3, -4$

$y(t) = Ae^{3t} + Be^{-4t}$

$y'(t) = 3Ae^{3t} - 4Be^{-4t}$

$y(0) = 3 = A + B$

$y'(0) = -4 = 3A - 4B$

$3(3-B) - 4B = -4$

$13 = 7B$

$B = 13/7$

$A = 3 - \frac{13}{7} = \frac{21}{7} - \frac{13}{7} = \frac{8}{7}$

$y(t) = \frac{8}{7}e^{3t} + \frac{13}{7}e^{-4t}$

Solve the initial value problem

$$y'' + y' - 12y = 0$$

$$y(0) = 3, y'(0) = -4$$

- Ⓐ $y(t) = 3e^{-4t} - 4e^{3t}$
- Ⓑ $y(t) = \frac{16}{7}e^{-3t} + \frac{5}{7}e^{4t}$
- Ⓒ $y(t) = \frac{13}{7}e^{-4t} + \frac{8}{7}e^{3t}$
- Ⓓ $y(t) = -\frac{5}{7}e^{-4t} - \frac{16}{7}e^{3t}$
- Ⓔ $y(t) = \frac{11}{7}e^{-3t} + \frac{10}{7}e^{4t}$

Find the general solution to the homogenous differential equation using the substitution $v = y/x$. Assume $x > 0$.

$$x^2 y' = xy + x^2 e^{y/x}$$

- Ⓐ $y = -x\ln(C - \ln x)$
- Ⓑ $y = x(\ln x + C)$
- Ⓒ $y = -\ln(C - \ln x)$
- Ⓓ $y = x\ln\left(\frac{x^2}{2} + C\right)$
- Ⓔ $y = -x\ln(C - x)$

$v = y/x$

$y = xv$

$dy/dx = v + x\,dv/dx$

$$\frac{dy}{dx} = \frac{y}{x} + e^{y/x}$$

$$v + x\frac{dv}{dx} = v + e^v$$

$$x\,dv/dx = e^v$$

$$\int e^{-v}\,dv = \int x^{-1}\,dx$$

$$-e^{-v} = \ln x + C$$

$$-e^{-y/x} = \ln x + C$$

$$e^{-y/x} = -C - \ln x = C - \ln x$$

$$-\frac{y}{x} = \ln(C - \ln x)$$

$$y = -x\ln(C - \ln x)$$

Find a linear homogenous constant-coefficient differential equation with the general solution $y(x) = C_1 e^{2x} + C_2 \cos(2x) + C_3 \sin(2x)$.

let $C_1, C_2, C_3 = A, B, C$

$$y(x) = Ae^{2x} + e^{0x}\left[B\cos 2x + C\sin 2x\right]$$

$0 \pm j2$

$r = 2, \pm j2$

$$(r-2)(r^2+4) = 0$$

$$r^3 + 4r - 2r^2 - 8 = 0$$

$$y''' - 2y'' + 4y' - 8y = 0$$