$M(x,y)dx + N(x,y)dy = 0$

$\frac{\partial F}{\partial x} = M, \quad \frac{\partial F}{\partial y} = N$

$(3x^2 + y - 4)dx - (2y - x)dy = 0$

$3x^2 + y - 4 \quad dy = 1$

$2y - x \quad dx = -1$

eqn. is exact

The general solution of the following differential equation

$$(3x^2 + y - 4) - (2y - x)\frac{dy}{dx} = 0$$

is

Ⓐ $x^3 + xy - 4x - c = 0$

Ⓑ $x^3 + 2xy - c = 0$

Ⓒ $x^3 + 2xy + y - c = 0$

Ⓓ $x^3 + 2xy - 4x - 2y^2 - c = 0$

Ⓔ $x^3 + xy - 4x - y^2 - c = 0$

$F = \int 3x^2 + y - 4 \, dx = x^3 + xy - 4x + g(y)$

$\frac{\partial F}{\partial y} = x^3 + xy - 4x + g(y) \rightarrow x + g(y) = N$

$x + g(y) = x - 2y \longrightarrow g(y) = -2y$

$\int g(y)\, dy = -y^2 + C$

$0 = x^3 + xy - 4x - y^2 + C$

$$\frac{y}{x} + 6x + (\ln x - 2y)\frac{dy}{dx} = 0$$

$$\left(\frac{y}{x} + 6x\right)dx + (\ln x - 2y)\,dy = 0$$

$$F = \int \frac{y}{x} + 6x\, dx = y\ln|x| + 3x^2 + g(y)$$

$$\frac{\partial F}{\partial y} = y\ln|x| + 3x^2 + g(y) = \ln x + g'(y)$$

$$\ln x + g'(y) = \ln x - 2y \rightarrow g'(y) = -2y$$

$$g(y) = \int -2y\, dy = -y^2$$

$$C = y\ln|x| + 3x^2 - y^2$$

$$C = 2\ln 1 + 3 - 4 = -1$$

If the following differential equation is exact, select the implicit solution to the initial value problem

$\left(\frac{y}{x} + 6x\right) + (\ln x - 2y)y' = 0 \qquad y(1) = 2; \qquad x > 0.$

If it is not exact, select "NOT EXACT".

- Ⓐ $y^2/x - 6xy + y\ln x = -8$
- Ⓑ $y\ln x - 3x^2 + y^2 = 1$
- Ⓒ $y\ln x - 4xy = -8$
- Ⓓ $y\ln x + 3x^2 - y^2 = -1$
- Ⓔ NOT EXACT

$$\frac{dy}{dx} = -\frac{y}{x} + \frac{2}{x^2 y} \rightarrow x^2 y\, dy = -xy^2 + 2\, dx$$

$$(xy^2 - 2)dx + x^2 y\, dy = 0$$

The solution of

$$y' + \frac{y}{x} = \frac{2}{x^2 y}, \quad x \neq 0$$

is given by

Ⓐ $x^2y^2 + 4xy = C$

Ⓑ $x^2y^2 + 4x = C$

Ⓒ $xy^2 - 2x = C$

Ⓓ $x^2y^2 - 4x = C$

Ⓔ $xy^2 - 4x = C$

$$xy^2 - 2\, dy = 2xy$$

$$x^2 y\, dx = 2xy$$

$$F = \int xy^2 - 2\, dx = \frac{1}{2}x^2y^2 - 2x + g(y)$$

$$\frac{\partial F}{\partial y} = \frac{1}{2}x^2y^2 - 2x + g(y) \rightarrow \partial F = \frac{1}{2}x^2y^2 - 2x + g(y)\, \partial y$$

$$M = x^2 y + g'(y) = x^2 y \rightarrow g'(y) = 0$$

$$g(y) = 0 + C$$

$$C = \frac{1}{2}x^2y^2 - 2$$

$2x + 7y \; dy = 7$

$7x + 8y \; dx = 7$

$F = \int M \, dx = \int 2x + 7y \, dx$

$= x^2 + 7xy + g(y)$

$\frac{\partial F}{\partial y} = x^2 + 7xy + g(y) \rightarrow \partial F = 7x + g'(y)$

$\partial F = N = 7x + g'(y) = 7x + 8y$

$g'(y) = 8y \rightarrow g(y) = 4y^2$

$C = x^2 + 7xy + 4y^2$

Which of the following is an implicit solution to the differential equation

$$(2x + 7y)dx + (7x + 8y)dy = 0$$

- (A) $x^2 + 7xy + 4y^2 = C$
- (B) $x^2 - 7xy - 4y^2 = C$
- (C) $x^2 + 4y^2 = C$
- (D) $x^2 - 4y^2 = C$
- (E) $x^3 - 7xy + 4y^2 = C$

$$y\cos 3xy + bx \; dy = bx\cos 3xy - 2y \; dx$$

$$\cos 3xy - 3xy\sin 3xy = b\cos 3xy - 3bxy\sin 3xy$$

$$\frac{\cos 3xy - 3xy\sin 3xy}{\cos 3xy - 3xy\sin 3xy} = 1 = b$$

$$F = \int y\cos 3xy + x \; dx = \frac{1}{3}\sin 3xy + \frac{1}{2}x^2 + g(y)$$

$$N = x\cos 3xy - 2y = \frac{1}{3}\sin 3xy + \frac{1}{2}x^2 + g(y) \; dy$$

$$= x\cos 3xy - 2y = x\cos 3xy + g'(y)$$

$$g'(y) = -2y \rightarrow g(y) = -y^2 + C$$

$$C = \frac{\sin 3xy}{3} + \frac{x^2}{2} - y^2$$

Find the value of b for which the given equation is exact, and then solve it using that value of b.

$$(y\cos(3xy) + bx)\,dx + (bx\cos(3xy) - 2y)\,dy = 0$$

Ⓐ $\cancel{b = -2,\ -\frac{2\sin(3xy)}{3} - y^2 - x^2 = C}$

Ⓑ $b = 1,\ \frac{\cos(3xy)}{9y^2} + \frac{x\sin(3xy)}{3y} - y^2 + xy = C$

Ⓒ $\cancel{b = 3,\ \sin(3xy) - y^2 + \frac{3x^2}{2} = C}$

Ⓓ $b = 1,\ \frac{\sin(3xy)}{3} - y^2 + \frac{x^2}{2} = C$

Ⓔ $\cancel{b = 3,\ \frac{\sin(3xy)}{3} - y^2 + \frac{3x^2}{2} = C}$

Find the value of the parameter α for which the equation is exact and then find an implicit solution for the initial value problem

$$(2xy^2 + x^2)dx + (\alpha x^2 y + 4y^3)dy = 0$$

$$y(0) = 2$$

$$2xy^2 + x^2 \, dy = \alpha x^2 y + 4y^3 dx$$

$$4xy = 2\alpha xy$$

$$4 = 2\alpha \rightarrow \alpha = 2$$

$$F = \int 2xy^2 + x^2 \, dx = x^2y^2 + \tfrac{1}{3}x^3 + g(y)$$

$$\frac{\partial F}{\partial y}\left(x^2y^2 + \tfrac{1}{3}x^3 + g(y)\right) = N = 2x^2y + 4y^3 = 2x^2y + g'(y)$$

$$g'(y) = 4y^3 \rightarrow g(y) = y^4$$

$$C = x^2y^2 + \tfrac{1}{3}x^3 + y^4$$

$$C = 16$$

$$16 = x^2y^2 + \tfrac{1}{3}x^3 + y^4$$

Which of the following is an implicit solution to the initial value problem

$$(\pi e^{\pi x} - y + \sin y)\,dx + (x\cos y - x + e^{-y})\,dy = 0, \quad y(1) = -\pi.$$

Ⓐ $x\sin y - yx + e^{\pi x} + e^{-y} = \pi + 2e^{\pi}$

Ⓑ $x\sin y - e^{-y} + e^{\pi x} - yx = \pi$

Ⓒ $x\cos y - xy - e^{-y} + \pi e^{\pi x} = (e^{\pi} + 1)(\pi - 1)$

Ⓓ $e^{\pi x} - xy + x\sin y - e^{-y} = -\pi$

Ⓔ $e^{\pi x} - xy - x\cos y - e^{-y} = \pi + 1$

$$\pi e^{\pi x} - y + \sin y \quad dy = x\cos y - x + e^{-y} \quad dx$$

$$-1 + \cos y = \cos y - 1$$

$$F = \int \pi e^{\pi x} - y + \sin y \, dx = e^{\pi x} - yx + x\sin y + g(y)$$

$$\frac{\partial F}{\partial y} = -x + x\cos y + g'(y) = N = x\cos y - x + e^{-y}$$

$$g'(y) = e^{-y}$$

$$g(y) = \int e^{-y}\,dy = -e^{-y} + C$$

$$C = e^{\pi x} - yx + x\sin y - e^{-y}$$

$$C = \cancel{e^{\pi}} + \pi + \sin - \pi \cancel{- e^{\pi}}$$

$$C = \pi$$

$$\pi = e^{\pi x} - yx + x\sin y - e^{-y}$$

$$y^2-1 \quad dy \overset{?}{=} 2xy+3y^2 \, dx$$

$$2y = 2y \checkmark$$

$$F = \int y^2-1 \, dx + g(y) = xy^2 - x + g(y)$$

$$\frac{\delta F}{\delta y} = 2xy + g'(y) = 2xy + 3y^2$$

$$3y^2 = g'(y) \rightarrow g(y) = y^3$$

$$C = xy^2 - x + y^3$$

$$C = (0)(2^2) - 0 + 2^3 = 8$$

$$8 = xy^2 - x + y^3$$

The solution of the initial value problem

$$(y^2-1)\,dx + (2xy+3y^2)\,dy = 0, \quad y(0)=2$$

is given implicitly by

Ⓐ $\frac{y^3}{3} - y + xy^2 + y^3 = \frac{24}{3}$

Ⓑ $xy^2 - x + y^2 = 4$

Ⓒ $xy^2 - x + y^3 = 8$

Ⓓ $x^2y + 3x - y^2 = 4$

Ⓔ $3xy^2 - x + y^3 = 8$

Find the value of the parameter α for which the equation is exact and then find an implicit solution of the initial value problem

$$(\alpha xy + 5y)dx + (x^2 + 5x - \sin y)dy = 0, \quad y(1) = 0.$$

$\alpha xy + 5y \, dy = x^2 + 5x - \sin y \, dx$

$\alpha x + 5 = 2x + 5 \rightarrow \alpha = 2$

$F = \int 2xy + 5y \, dx + g(y) = x^2y + 5xy + g(y)$

$\frac{\partial F}{\partial y} = x^2 + 5x + g'(y) = M = x^2 + 5x - \sin y$

$g'(y) = -\sin y \Rightarrow g(y) = \cos y + C$

$C = x^2y + 5xy + \cos y$

$C = (1)(0) + 5(1)(0) + \cos 0 = 1$

$1 = x^2y + 5xy + \cos y$

Explain why the following equation is exact and find an implicit formula for the solution to the initial value problem

$$(2xy - y^3)dx + (6y + x^2 - 3xy^2)dy = 0, \quad y(1) = 2.$$

$$2xy - y^3 \, dy \stackrel{?}{=} 6y + x^2 - 3xy^2 \, dx$$

$$2x - 3y^2 = 2x - 3y^2$$

The eqn. is exact because in this case $M\,dy = N\,dx$

$$F = \int 2xy - y^3 \, dx + g(y) = x^2y - xy^3 + g(y)$$

$$\frac{dF}{dy} = x^2 - 3xy^2 + g'(y) = N = 6y + x^2 - 3xy^2$$

$$g'(y) = 6y \Rightarrow g(y) = 3y^2 + C$$

$$C = x^2y - xy^3 + 3y^2$$

$$C = 1^2(2) - 1(2^3) + 3(2^2) = 2 - 8 + 12 = 6$$

$$6 = x^2y - xy^3 + 3y^2$$